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Strong (weak) domination in interval valued fuzzy graph

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ABSTRACT

Let G =(A,B) be an interval valued fuzzy graph on V and x,y∈V, We say x dominates y if $\mu_B^-(xy) = \min\{\mu_A^-(x), \mu_A^-(y)\}$and $\mu_B^+(xy) = \max\{\mu_A^+(x),\mu_A^+(y)\}$.A subset S of V is called a strong (weak) dominating set in interval valued fuzzy graph G if every vertex x∈V-S is strongly (weakly) dominated some vertex y in S. The minimum cardinality taken over all strong (weak) dominating set of interval valued fuzzy is called the strong (weak) domination number of G. we introduce strong (weak) domination number in interval valued fuzzy graphs and obtain some interesting results for this new parameter in interval valued fuzzy graphs.

Index terms: Dominating set, Domination in interval valued fuzzy graph, Fuzzy graphs, strong (weak) domination in interval valued fuzzy graph

1. INTRODUCTION

Zadeh [16] introduced the notion of interval-valued fuzzy sets as an extension of fuzzy sets which gives a more precise tool to model uncertainty in real life situations. Some recent work of Zadeh in connection with the importance of fuzzy logic may be found in [15]. Interval-valued fuzzy sets have been widely used in many areas of science and engineering, e g., in approximate reasoning medical diagnosis, multi valued logic, intelligent control, and topological spaces etc. Hongmeiand Lianhua introduced the definition of interval valued fuzz y graphs in [3]. Recently, Akram and Dudek [1] have studied several properties and operations on interval valued fuzzy graphs. In this paper, we analyze bounds on strong (weak) dominating set of interval valued fuzzy graph.

2. PRELIMINARIES

Throughout this paper a graph will denote a graph without loops. First we collect some definitions to be used in this paper.

Definition 2.1 An interval- valued fuzzy set A on a set V is defined by

A = {(x, [μ_A^-(x),μ_A^+(x]) :x∈V},

Where μ_A^- and μ_A^+ are fuzzy subsets of V such thatμ_A^-(x) ≤ μ_A^+(x) for all x ∈ V. If G $*$ = (V , E) is a crisp graph, then by an interval-valued fuzzy relation B on V we mean an interval-valued fuzzy set on E such that μ_B^-(xy) ≤ min {μ_A^-(x),μ_A^-(y)}and μ_B^+(xy) ≤ max{μ_A^+(x),μ_A^+(y)} for all xy∈ E and We write

B = {xy ,[μ_B^-(xy),μ_B^+(xy)]:xy∈E }

Definition 2. 2. An interval-valued fuzzy graph of a graph G $*$= (V , E) is a pair G = (A, B), where

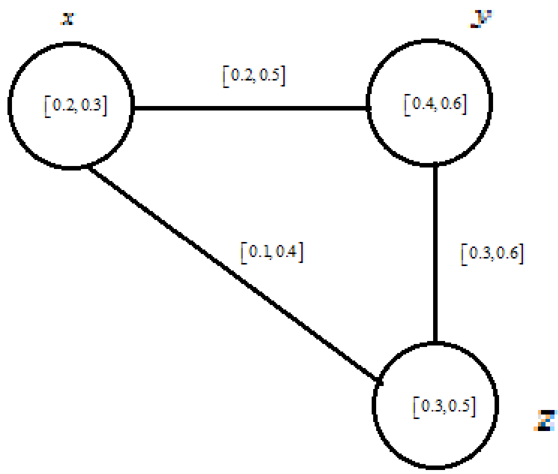
A = [μ_A^- ,μ_A^+] is an interval-valued fuzzy set on V and B =[μ_B^- ,μ_B^+]] is an interval-valued fuzzy relation on V

Example 2.3. Consider the graph G $*$ = (V , E), where V = { x, y , z } and E ={ xy , yz , zx}. Let A be an interval-valued fuzzy set on V and let B be an interval-valued fuzzy set on E ⊆ V × V defined by

$$A=\left\langle\left(\frac{x}{0.2},\frac{y}{0.4},\frac{z}{0.3}\right)\right\rangle,\left\langle\left(\frac{x}{0.3},\frac{y}{0.6},\frac{z}{0.5}\right)\right\rangle$$

$$B=\left\langle\left(\frac{xy}{0.2},\frac{yz}{0.3},\frac{zx}{0.1}\right)\right\rangle\left\langle\left(\frac{xy}{0.5},\frac{yz}{0.6},\frac{zx}{0.4}\right)\right\rangle$$

Then G = (A, B) is an interval-valued fuzzy graph of a graph G $*$ = (V , E)



Definition 2.4 The order p and size q of an interval-valued fuzzy graph of a graph G = (A, B) of a graph G * =(V , E) are defined to be

$$p = \sum_{v \in V} \frac{1 + \mu_A^+(v) - \mu_A^-(v)}{2}$$

And

$$q = \sum_{xy \in V} \frac{1 + \mu_B^+(xy) - \mu_B^-(xy)}{2}$$

Definition 2. 5. Let G = (A, B) be an interval-valued fuzzy graph of a graph on G * = (V , E) and S ⊆ V Then the cardinality of S is defined to be

$$\sum_{v \in V} \frac{1 + \mu_A^+(v) - \mu_A^-(v)}{2}$$

Definition 2. 6. An interval-valued fuzzy graph of a graph G = (A, B) of a graph G * = (V , E) is said to be complete if $\mu_B^-(xy) = \min\{\mu_A^-(x), \mu_A^-(y)\}$ and $\mu_B^+(xy) = \max\{\mu_A^+(x), \mu_A^+(y)\}$ for all xy∈ E and is denote d by $K_{\mu A}$

Definition 2. 7. The complement of an interval-valued fuzzy graph of a graph G of a graph G * = (V, E)is the interval-valued fuzzy graph of a graph $\bar{G} = (\bar{A}, \bar{B})$,

where $\bar{A} = [\mu_A^+, \mu_A^-]$ and $\bar{B} = [\bar{\mu}_B^-, \bar{\mu}_B^+]$ is defined by $\overline{\mu_B^-}(xy) = \min\{\mu_A^-(x), \mu_A^-(y)\} - \mu_B^-(xy)$

$\overline{\mu_B^+}(xy) = \max\{\mu_A^+(x), \mu_A^+(y)\} - \mu_B^+(xy)$ for all xy∈ E

Definition 2. 8. An interval-valued fuzzy graph of a graph G = (A, B) of a graph G * = (V , E) is said to be bipartile if the vertex set V can be partitioned into two non empty sets V $_1$ and V $_2$ such that μ_B^-(xy) = 0 and μ_B^+(xy) =0 if x,y∈V $_1$ or x,y∈ V $_2$. Further if μ_B^+(xy)=max{μ_A^+(x),μ_A^+(y)} and μ_B^-(xy) =min{μ_A^-(x),μ_A^-(y)} for all x ∈V $_1$ and y ∈ V $_2$ then G is called a complete bipartile graph and is denoted by $k_{\mu_A^-,\mu_A^+}$ where $\mu_A^-$and $\mu_A^+$are restrictions of $\mu_A^-$and μ_A^+on V $_1$ and V $_2$ respectively.

Definition 2.9. An edge e = xy of an interval-valued fuzzy graph of G is called an effective edge if μ_B^-(xy) =min {μ_A^-(x),μ_A^-(y)}andμ_B^+(xy) = max{μ_A^+(x),μ_A^+(y)}. In this case, the vertex x is called a neighbor of y and conversely. N (x) = {y ∈ V: y is a neighbor of x} is called the neighborhood of x.

Example 2.10. Consider the graph G * = (V , E), where V =

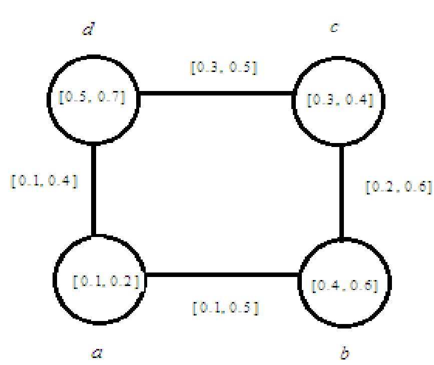
{ a,b,c,d} and E ={ ab,bc,cd,da}. Let A be aninterval- valued fuzzy set on V and let B be an interval-valued fuzzy set on

E ⊆ V × V defined by

$$A = \left\langle \left(\frac{a}{0.1}, \frac{b}{0.4}, \frac{c}{0.3}, \frac{d}{0.5} \right) \right\rangle, \left\langle \left(\frac{a}{0.2}, \frac{b}{0.6}, \frac{c}{0.3}, \frac{d}{0.7} \right) \right\rangle$$

$$B = \left\langle \left(\frac{ab}{0.1}, \frac{bc}{0.2}, \frac{cd}{0.3}, \frac{da}{0.1} \right) \right\rangle, \quad \left\langle \left(\frac{ab}{0.5}, \frac{bc}{0.6}, \frac{cd}{0.5}, \frac{da}{0.4} \right) \right\rangle$$

Then G = (A, B) is an interval-valued fuzzy graph of G *= (V, E)



In this example, ab and da are effective edges. Also,

N (a) = {b, d}, N (b) = {a}, N (d) = {a}, N (c) = φ (the empty set).

Definition 2.11. Let G = (A, B) be an interval-valued fuzzy graph on V and x, y ∈ V . We say x dominates y if $\mu_B^-(xy) = \min\{\mu_A^-(x), \mu_A^-(y)\}$ and $\mu_B^+(xy) = \max\{\mu_A^+(x), \mu_A^+(y)\}$

A subset S of V is called a dominating set in G if for every v ∉S , there existsu ∈ S such that u dominates v . The minimum cardinality of a dominating set in G is called the domination number of G and is denoted by γ (G).

Remark 2.12. (i) For any x, y ∈V , if x dominates y then y dominates x and as such domination is a symmetric relation.

(ii) $\mu_B^-(xy) < \min\{\mu_A^-(x), \mu_A^-(y)\}$ and $\mu_B^+(xy) < \max\{\mu_A^+(x), \mu_A^+(y)\}$x, y ∈V ,then the only dominating set in G is V.

Remark 2.13 (i) Since {v } is a dominating set of K $_{\mu A}$ for each

v∈ V , we have

$$(i)\gamma(k_{\mu A}) = \min_{v\in V} \frac{1+\mu_A^+(v)-\mu_A^-(v)}{2}$$

$$(iii)\gamma(k_{\mu A^1,\mu A^2}) = \min_{v\in V} \frac{1+\mu_A^+(v)-\mu_A^-(v)}{2}$$

$$(ii)\gamma(\overline{k_{\mu A}}) = p \quad + \min_{w\in V} \frac{1+\mu_A^+(w)-\mu_A^-(w)}{2}$$

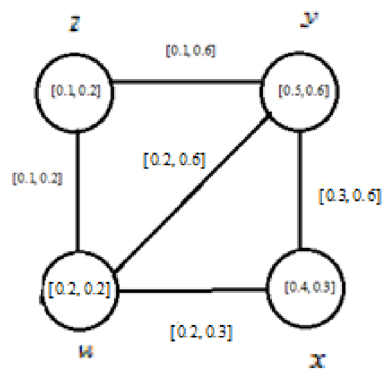
Example 2.14 Consider the graph G * = (V , E), where

V = { w,x,y,z} and E ={ wx,xy,yz,zw,wy}. Let A be an interval- valued fuzzy set on V and let B be an interval-valued fuzzy set on E ⊆ V × V defined by

$$A = \left\langle \left(\frac{w}{0.2}, \frac{x}{0.3}, \frac{y}{0.5}, \frac{z}{0.1} \right) \right\rangle \left\langle \left(\frac{w}{0.2}, \frac{x}{0.3}, \frac{y}{0.6}, \frac{z}{0.2} \right) \right\rangle B = \left\langle \left(\frac{wx}{0.2}, \frac{xy}{0.3}, \frac{yz}{0.1}, \frac{wy}{0.2}, \frac{zw}{0.1} \right) \right\rangle , \left\langle \left(\frac{wx}{0.3}, \frac{xy}{0.6}, \frac{yz}{0.6}, \frac{wy}{0.6}, \frac{zw}{0.2} \right) \right\rangle$$

Then G = (A, B) is an interval-valued fuzzy graph of

G * = (V, E).



Since {y} is a dominating set of G for each y ∈ V and

We have the domination number of interval- valued fuzzy graph G,

$$\gamma(G) = \min_{y \in V} \frac{1+\mu_A^+(y)-\mu_A^-(y)}{2} = \frac{1+0.6-0.5}{2} = 0.55$$

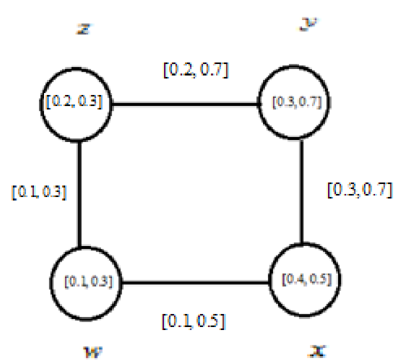
Example 2.15 Consider the graph G * = (V , E), where V = { w,x,y,z} and E ={ wx,xy,yz,zw }. Let A be aninterval-valued fuzzy set on V and let B be aninterval-valued fuzzy set on E ⊆ V × V defined by

$$A = \left\langle \left(\frac{w}{0.1}, \frac{x}{0.4}, \frac{y}{0.3}, \frac{z}{0.2} \right) \right\rangle, \left\langle \left(\frac{w}{0.3}, \frac{x}{0.5}, \frac{y}{0.7}, \frac{z}{0.3} \right) \right\rangle$$

$$B = \left\langle \left(\frac{wx}{0.1}, \frac{xy}{0.3}, \frac{yz}{0.2}, \frac{zw}{0.1} \right) \right\rangle, \left\langle \left(\frac{wx}{0.5}, \frac{xy}{0.7}, \frac{yz}{0.7}, \frac{zw}{0.3} \right) \right\rangle$$

Then G = (A, B) is an interval-valued fuzzy graph of

G * = (V , E).



Since {w,y} is a minimal dominating set of G for each w,y∈ V and We have the domination number of interval- valued fuzzy graph G,

$$\gamma(G)=\min_{w\in V}\frac{1+\mu_A^{+}(w)-\mu_A^{-}(w)}{2}+\min_{y\in V}\frac{1+\mu_A^{+}(y)-\mu_A^{-}(y)}{2}$$

$$=\frac{1+0.3-0.1}{2}+\frac{1+0.7-0.3}{2}=1.3$$

3. STRONG (WEAK) DOMINATION IN INTERVAL VALUED FUZZY GRAPH

In the section, we introduce strong (weak) domination in interval valued fuzzy graph.

Definition 3.1 Let G = (A,B) be an interval valued fuzzy graph on V and x,y∈V, Then u strongly dominates v(v weakly dominates u) if

i. $\mu_B^-(xy)=\min\{\mu_A^-(x),\mu_A^-(y)\}$ and $\mu_B^+(xy)=\max\{\mu_A^+(x),\mu_A^+(y)\}$

ii. $d_N(x)\geq d_N(y)$

Definition 3.2A subset S of V is called a strong (weak) dominating set of interval valued fuzzy graph G if every vertex x∈V-S is strongly (weakly) dominated some vertex y in S. The minimum cardinality taken over all strong (weak) dominating set of interval valued fuzzy is called the strong (weak) domination number of G and it is denoted by $\gamma_s(G)(\gamma_w(G))$

Example 3.3 Consider the graph G * = (V , E), where

V = { a,b,c,d,e} and E ={ ab,ac,ad,ae,bc,cd,de }.

Let A be an interval- valued fuzzy set on V and let

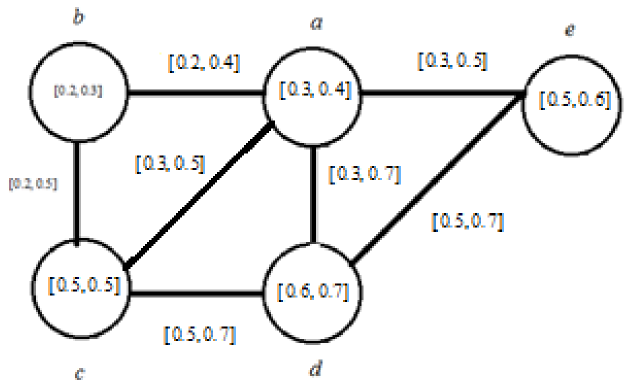
B be aninterval-valued fuzzy set on E ⊆ V × V defined by

$$A=\left\langle\left(\frac{a}{0.3},\frac{b}{0.2},\frac{c}{0.5},\frac{d}{0.6},\frac{e}{0.5}\right)\right\rangle,\left\langle\left(\frac{a}{0.4},\frac{b}{0.3},\frac{c}{0.6},\frac{d}{0.7},\frac{e}{0.6}\right)\right\rangle$$

$$B=\left\langle\left(\frac{ab}{0.2},\frac{ac}{0.3},\frac{ad}{0.3},\frac{ae}{0.3},\frac{bc}{0.2},\frac{cd}{0.5},\frac{de}{0.5}\right)\right\rangle,\left\langle\left(\frac{ab}{0.4},\frac{ac}{0.5},\frac{ad}{0.7},\frac{ae}{0.6},\frac{bc}{0.5},\frac{cd}{0.7},\frac{de}{0.7}\right)\right\rangle$$

Then G = (A, B) is an interval-valued fuzzy graph of

G * = (V , E).



Since {a} is a strong dominating set of G for each

a∈ V and {c,e} is a minimal weak dominating set of G

for each c,e∈ V. We have the strong,Weak domination number in interval- valued fuzzy graph G,

$$\gamma_S(G) = \min_{a \in V} \frac{1+\mu_A^+(a)-\mu_A^-(a)}{2} = \frac{1+0.4-0.3}{2} = 0.55 \quad \gamma_w(G) = \min_{c \in V} \frac{1+\mu_A^+(c)-\mu_A^-(c)}{2} + \min_{e \in V} \frac{1+\mu_A^+(e)-\mu_A^-(e)}{2}$$

$$= \frac{1+0.5-0.5}{2} + \frac{1+0.6-0.5}{2} = 1.05$$

Theorem 3.4 Let D be a minimal strong dominating set of interval- valued fuzzy graph G .Then for each u∈D of the following holds.

(i) No vertex in D strongly dominate v

(ii) There exists v∈ V-D such that v is the only vertex in D which strongly dominates u.

Proof :

Suppose D is a minimal dominating set of G. Then for each node u∈D the set D′=D-{u} is not a dominating set. Thus, there is a node v∈V-D′ which is not dominated by any node in D′. Now either u=v or v∈V-D, If v=u then no vertex in D strongly dominates v. If v∈V-D and v is not dominated by D-{u} but is dominated by D, Then u is the only strong neighbor of v and v is the only vertex in D which strongly dominates u.

Conversely suppose D is a dominating set and each node u∈D, one of the two stated conditions holds. Now we prove D is a minimal strong dominating set. Suppose D is not a minimal strong dominating set, then there exists a node u∈D such that D-{u} is a dominating set. Therefore condition (i) does not hold. Also if D-{u} is a dominating set then every node in V-D is a strong neighbor to at least one node in D-{u}. Therefore condition (ii) does not hold. Hence neither condition (i) nor (ii) holds which is a contradiction.

Theorem 3.5 For a interval- valued fuzzy graph G of order p,**(**i) $\gamma(G) \leq \gamma_s(G) \leq P\text{-}\Delta_N(G) \leq p\text{-}\Delta_E(G)$

(ii) $\gamma(G) \leq \gamma_w(G) \leq P\text{-}\delta_N(G) \leq p\text{-}\delta_E(G)$ where $\Delta_N(G)$ $[\Delta_E(G)]$ and $\delta_N(G)$ $[\delta_E(G)]$ denote the maximum and minimum neighborhood degrees (effective degrees) of G

Proof

Since every strong Weak) dominating set is a dominating set of G,$\gamma(G) \leq \gamma_s(G)$ and$\gamma(G) \leq \gamma_w(G)$.Let u ,v$\in$V, If $\mu_B^-(xy)$ =min{$\mu_A^-(x)$,$\mu_A^-(y)$} and $\mu_B^+(xy)$ = max{$\mu_A^+(x)$,$\mu_A^+(y)$},$d_{N(u)} = \Delta_N(G)$ and $d_{N(v)} = \delta_N(G)$. Then clearly V-N(u) is a strong dominating set and V-N(v) is weak dominating set. Therefore $\gamma_s(G) \leq |V\text{-}N(u)|$ and $\gamma_w(G) \leq |V\text{-}N(u)|$ that is $\gamma_s(G) \leq P\text{-}\Delta_N(G)$ and $\gamma_w(G) \leq P\text{-}\delta_N(G)$ Further since $\Delta_E(G)] \leq \Delta_N(G)$and $\delta_E(G) \leq \delta_N(G)$Hence $\gamma(G) \leq \gamma_s(G) \leq P\text{-}\Delta_N(G) \leq p\text{-}\Delta_E(G)$ and $\gamma(G) \leq \gamma_w(G) \leq P\text{-}\delta_N(G) \leq p\text{-}\delta_E(G)$.

4. CONCLUSION

Strong (weak) domination in interval valued fuzzy graph is defined. Theorems related to this concept are derived and the relation between domination number in interval valued fuzzy graph and Strong (weak) domination in interval valued fuzzy graphs are established.

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